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FILM COOLING FROM A SINGLE HOLE AND A ROW OF HOLES OF VARIABLE PITCH TO DIAMETER RATIO

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Abstract—Measurements of film cooling effectiveness are presented for coolant injection through a single hole and rows of holes with pitch to diameter ratios of 8.0, 5.33 and 2.67 with an injection angle of 30° to the free stream flow direction. In the case of a row of holes with a pitch to diameter ratio of 5.33 the influence on film cooling of injecting a small fraction of the coolant through narrow slits located three diameters downstream from the row of holes is investigated and is found to be significant. Also the effects of free stream turbulence and velocity gradient are measured and in general increased free stream turbulence reduces film cooling effectiveness, whereas, the influence of free stream velocity gradient differs for two and three dimensional flow situations. The measurements are analysed via line sink models and correlations based on an energy balance, the latter are much more successful in describing the film cooling effectiveness measurements.

NOMENCLATURE

A_h,	$= (Ms_h/X)(Re_x)_\infty^{0.2}$, correlation group for a single hole;
A_r,	$= (Ms_r/X)(Re_x)_\infty^{0.2}$, correlation group for a row of holes;
D,	hole diameter;
I,	$= \rho_c U_c^2/\rho_\infty U_\infty^2$, momentum flux ratio;
M,	$= \rho_c U_c/\rho_\infty U_\infty$, blowing parameter;
M',	$= MD/P$, modified blowing parameter;
P,	pitch of holes;
Re_D,	coolant Reynolds number;
$(Re_x)_\infty$,	free stream Reynolds number;
s_h,	$= D(\pi)^{1/2}/2$, equivalent slit opening of a single hole;
s_r,	$= D^2\pi/4P$, equivalent slit opening of a row of holes;
T_c,	inlet coolant temperature;
$T_{(x,y,z)}$,	temperature at location x, y, z;
T_∞,	free stream temperature;
U_c,	inlet coolant velocity;
U_∞,	free stream velocity;
U'/U_∞,	turbulence intensity;
V,	$= (\nu/U_\infty^2)(\mathrm{d}U_\infty/\mathrm{d}x)$, free stream velocity gradient factor;
x,	streamwise distance from the leading edge of the surface;
x_i,	streamwise distance from the leading edge of the surface of the injection holes;
X,	streamwise distance downstream from the centre of the holes;
Y,	distance above the surface;
Y_0,	distance above the surface to which the jet rises;
Z,	lateral distance from the centre of the holes.

Greek symbols

ε,	effective turbulent diffusivity;
η,	$= (T_\infty - T_{(x,0,z)})/(T_\infty - T_c)$, film cooling effectiveness
$\bar{\eta}$,	laterally averaged effectiveness over a pitch at a given downstream location;
ν,	kinematic viscosity;
ρ_c,	inlet coolent density;
ρ_∞,	free stream density.

INTRODUCTION

FILM cooling by injection through discrete holes is a promising technique for protecting the blades of the high pressure stages of gas turbines. Successes have been achieved with the film cooling technique, however, the performance of discrete hole cooling depends heavily on adopting the correct spacing between holes and the value of injectant velocity relative to the free stream velocity. Also the cooling performance depends on free stream, injectant and boundary-layer properties.

During the past decade the attentions of some researchers have been directed to discrete hole film cooling. Goldstein *et al.* [1] investigated the effect of angled injection through a discrete hole angled at 90 and 35° to the free stream flow direction. Film cooling from a circular hole with lateral inclination of 15 and 35° to the free stream flow direction was studied by Goldstein *et al.* [2] and a study by Hartsel [3] revealed the aerodynamic penalties of lateral angled injection on both suction and pressure surfaces of aerofoils. The improved cooling obtained by widening the exits from cooling holes was observed by Goldstein *et al.* [4]. Kruse [5] showed that a double row of holes provided better cooling than a single row, and in a study of multiple rows of film cooling holes Le Brocq *et al.* [6] concluded that a staggered pattern of holes gave superior cooling than holes in line. Sellers [7] proposed that the cooling effect of rows of holes could be predicted by taking into consideration the effectiveness level from

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each upstream injection location individually. This has been confirmed by Muska *et al.* [8] and Mayle and Camarata [9].

Very limited work has been carried out on the influence of free stream turbulence intensity on discrete hole film cooling. Launder and York [10] reported a drop in film cooling effectiveness due to increased free stream turbulence intensity in the presence of a favourable velocity gradient. Brown and Minty [11] reported the same for film cooling from a slot of aspect ratio 2 in the presence of adverse, favourable and zero velocity gradients. The effect of free stream velocity gradients on discrete hole film cooling has been investigated by Lander *et al.* [12], Launder and York [10], Nicolas and LeMeur [13], Brown and Minty [11] and Liess [14] but no consistency of view emerges from these works.

The flow field associated with injection through a discrete hole is three dimensional in nature. Experimental studies by Ambramovich [15], Keffer and Baines [16], Ramsey and Goldstein [17] and Bergeles *et al.* [18, 19] with angled injection have revealed the existence of streamwise vortices and a region of recirculation downstream of the point of injection. When the injection geometry involves a number of holes, the character of the flow becomes more complex as a consequence of the interaction between injectant jets. Bergeles *et al.* [19] reported the application of a three dimensional finite difference partially parabolic numerical procedure developed by Pratap and Spalding [20] to the prediction of mean velocity, pressure and temperature fields following injection through a single row of holes normal to and inclined at 45 and 35° to the free stream flow direction and for multiple rows of holes in staggered pattern with holes normal to the free stream flow direction. Also, they made predictions for a row of holes inclined at 30° to the free stream flow direction in the presence of a strong streamwise favourable velocity gradient.

Heat sink models for discrete hole injection started from the work of Ramsey *et al.* [21] who developed a relationship for film cooling effectiveness on an adiabatic surface. Evaluation of effectiveness requires specification of the effective turbulent diffusivity ε. Values of the lateral spreading of effectiveness for normal injection and $M = 0.1$, measured by Goldstein *et al.* [1] and Ramsey and Goldstein [17], were used to evaluate ε. It was found that the predicted values of effectiveness were much greater than the measured values near the injection hole but quite good agreement was obtained for $X/D > 10$. The predictions for larger blowing rates were in poor agreement with experiment, this occurs as the thermal diffusivity in the vertical direction is greater than in the lateral direction, but the model could not account for this. Using values of diffusivity obtained from experimental measurements of effectiveness immediately downstream of the point of injection underpredicted effectiveness far downstream. At higher blowing rates ($M \geqslant 0.4$) the injected jet lifts off the surface and Eriksen *et al.* [22] accounted for this by locating the point heat sink a distance $Y = Y_0$ above the injection hole. In order to satisfy the condition of an adiabatic wall at the $Y = 0$ plane an image sink was assumed at $-Y_0$ and the effectiveness calculated from the combined sinks. Eriksen *et al.* [22] proposed that Y_0 be taken as the average value of the distance above the surface in the injectant jet at which maximum difference in temperature between injectant and free stream occurs. In the same paper they extended the above model by assuming a line sink of length D extending in the Z direction and located Y_0 above the injection hole. The strength of the line sink per unit length was assumed constant.

The point sink model of Eriksen *et al.* [22] predicted centre line, that is, $Z = 0$, effectiveness results well but underestimated the spread of the jet and was poor for predicting effectiveness at lateral locations. The uniform strength line sink model showed an improvement in predicting off centre line effectiveness but was inferior to the point sink model on centre line. In view of this conflict Saluja [23] considered the use of line sink models of varying strength, firstly a line sink the strength of which varied linearly in the lateral direction and secondly a line sink the strength of which varied parabolically in the lateral direction. In both cases the strength was maximum on the centre line and zero at the edges of the injectant hole. A third model considered by Saluja [23] for a row of holes was a line sink model of uniform strength covering the complete pitch of holes. Comparisons of the sink models with experiment are made later in this article.

Correlations have been developed to describe film cooling measurements for continuous slot and single hole injection. Most of the correlations are based on the de-icing work of Wieghardt [24] or the two dimensional flow model for injection of Hatch and Papell [25] and Papell [26]. Brown [27] extended the work of Hatch and Papell [25] and Papell [26] to allow for exchange between the two dimensional jet and the free stream via an energy balance. From the two dimensional flow model Brown [27] suggested that for a given angle, slot opening and injected air to free stream velocity ratio, the measurements can be made to yield a single curve if a function of the momentum flux ratio ($I = \rho_c U_c^2/\rho_\infty U_\infty^2$) of the injectant to free stream is introduced as a multiplying factor to the correlating group. The multiplying factor was to account for the initial mixing of the free stream and injectant at the point of injection. Such correlations fitted well the two dimensional injection measurements of Brown [27] for 90, 60 and 45° angles of injection and Artt *et al.* [28] used the same relationships to correlate their measurements for a two dimensional injection geometry for 30 and 15° angle of injection. Brown and Minty [11] presented results of a study of film cooling through slots of various aspect ratios (24, 8, 4, 2, and 1) angled at 30°

to the free stream flow direction so that two and three dimensional injection geometries were investigated. The correlations of Brown [27] adequately described the results for aspect ratios 24 and 8 but for aspect ratios 4, 2 and 1 the correlations had to be modified.

EXPERIMENTAL RIG AND EXPERIMENTATION

The experimental investigation was carried out in a low speed recirculatory wind tunnel which provided a uniform free stream at controlled velocity up to 15 m/s and temperature up to 60°C. Cold air was injected through the base of the working section. Temperature measurements were made on the base of the working section downstream of the cold air injection tubes in the streamwise and lateral directions and velocity and turbulence measurements were made in the exit region of the tubes and in the boundary-layers, jets and free stream. The injection configurations investigated were: a single hole and a single row of holes with variable pitch to diameter ratios of 8.0, 5.33, 4.0, 3.2 and 2.67 inclined at 30° to the free stream flow direction, and the single row of holes with pitch to diameter ratio of 5.33 with a normal narrow slit located three diameters downstream from the holes trailing edges with the slit continuous and then plugged to create a single multiple slit row. The effects of free stream turbulence intensity up to values of 0.12, and favourable and adverse velocity gradients of values $+1.14 \times 10^{-6}$ and -0.58×10^{-6} respectively on film cooling were investigated for injection through a row of holes having a pitch to diameter ratio of 5.33. A plan view of the working section of the experimental rig for the slit arrangements used with a row of holes having $P/D = 5.33$ is illustrated in Fig. 1.

In the absence of cold air injection the free stream velocity at the point of injection was 10 m/s throughout the measurements described in this article, giving a value of 3.55×10^5 for the free stream Reynolds number at the point of cold air injection. The thickness of the boundary layer at the point of injection was 18 mm and so its ratio to the injection holes' diameter was 0.925 where $D = 19.4$ mm. The length to diameter ratio of the cold air injection tubes was 14.74. During film cooling measurements the temperature of the free stream was maintained about 30°C above the temperature of the injected coolant and the ratio of coolant to free stream density ρ_c/ρ_∞ was about 1.1. The velocity ratio U_c/U_∞ was variable up to a value of 1.2 and so the blowing rate $M = \rho_c U_c/\rho_\infty U_\infty$ was variable up to 1.35. The corresponding upper limit for the momentum flux ratio $I = \rho_c U_c^2/\rho_\infty U_\infty^2$ was about 1.6. The major portion of the work described in this article was for zero free stream velocity gradient and a turbulence intensity of about 0.017.

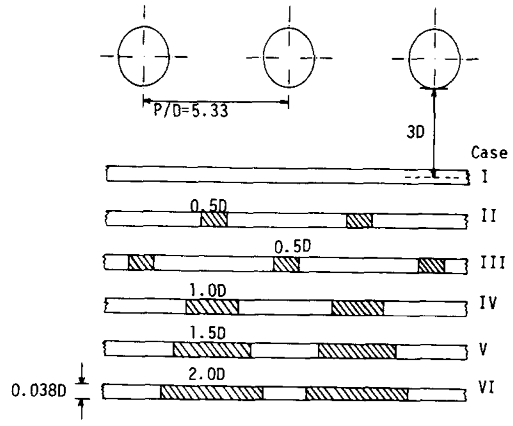

FIG. 1. Slit arrangements with a row of holes having $P/D = 5.33$ (not to scale).

In addition to temperature measurements for determination of adiabatic effectiveness microminiature hot wires were used to determine velocities and turbulence intensities in boundary layers, jets and the free stream, and flow visualisation techniques were also employed. The flow visualisation technique which proved most useful was the use of Cholesteric liquid crystals applied to the rig base surface behind the row of holes to give temperature patterns in the XZ plane at $Y = 0$, and the patterns obtained on a thin plate normal to the XZ plane and parallel to the XY plane and cutting centrally along the X direction and immersed into a cold air injection hole, the latter was useful in determining values of Y_0 for the heat sink models, Brown and Saluja [29].

FILM COOLING EFFECTIVENESS RESULTS

The effectiveness measurements for a single hole and a row of holes with pitch to diameter ratios 8.0, 5.33 and 2.67 for various values of M, X/D, Z/D and Z/P are presented in this article. The measurements for pitch to diameter ratios of 4.0 and 3.2 are not included in this article as they fall between those for pitch to diameter ratios of 5.33 and 2.67. The centre line and averaged effectivenesses for the row of holes with pitch to diameter ratios of 8.0 and 2.67 are illustrated in Figs. 2 and 3, the measurements for $P/D = 5.33$ are intermediate between $P/D = 8.0$ and 2.67 and have been omitted. Also incorporated in Fig. 2 is the variation of centre line effectiveness as measured by Goldstein *et al.* [2] for a row of holes with pitch to diameter ratio of 3.0 inclined at 35° to the free stream flow direction at $X/D = 6.7$ and $Re_D = 2.2 \times 10^4$. Some of the averaged effectiveness results for a row of holes with pitch to diameter ratio of 2.67 are replotted in Fig. 4 where they can be compared with the averaged effectiveness measurements of Goldstein *et al.* [2], Liess and Carnel [30] and Kruse [5]. A modified blowing parameter $M' = MD/P$ may be more meaningful than M as an indicator of averaged film cooling performance and so some of the averaged effectiveness results of Fig. 3 were replotted against M' by Saluja [23]. The modified blowing parameter normalises the injectant mass flow flux. The trends shown in Fig. 3 were reinforced when $\bar{\eta}$ was plotted against M'.

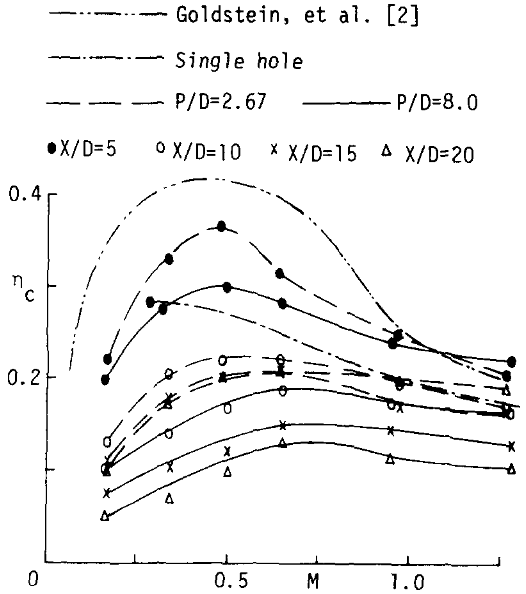

Fig. 2. η_c vs M for fixed values of X/D for rows of holes with $P/D = 2.67$ and 8.0.

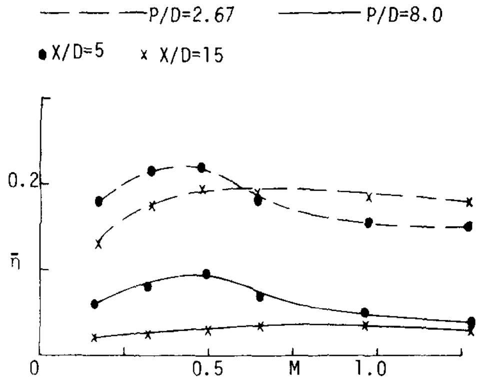

Fig. 3. $\bar{\eta}$ vs M for fixed values of X/D for rows of holes with $P/D = 2.67$ and 8.0.

The experimental arrangement with a row of holes of pitch to diameter ratio 5.33 was used to illustrate the influence of free stream turbulence intensity and velocity gradient on film cooling and these measurements are presented in Figs. 5 and 6. This arrangement was also used to illustrate the influence of injecting a small fraction (about 5%) of the total coolant through a continuous slit or a row of slits located three diameters downstream from the holes on film cooling from the row of holes. These measurements are illustrated in Figs. 7 and 8 where the key to the slit configurations is given in Fig. 1. Comparing cases I and VI with the measurements without a slit shows the range of influence of the modification.

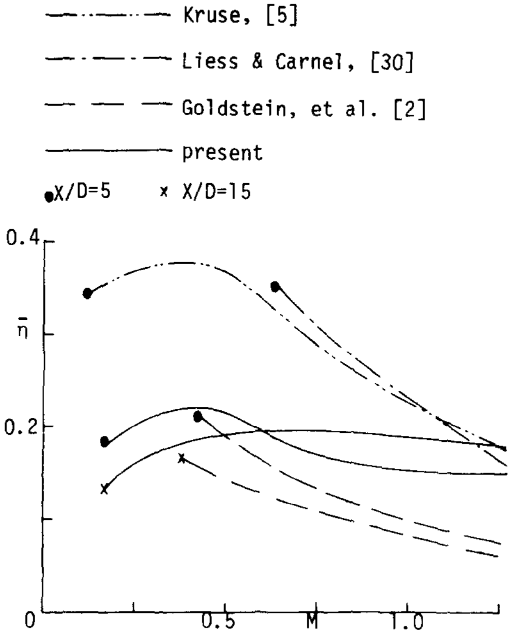

Fig. 4. Comparison of averaged results with other workers for a row of holes with $P/D = 2.67$.

DISCUSSION AND CONCLUSIONS ON THE MEASUREMENTS

The effect of decreasing the pitch to diameter ratio for injection through a row of holes from 8.00 through 5.33 down to 2.67 is apparent from Figs. 2 and 3, and also from Figs. 5–8. Important observations can be made from the various effectiveness measurements and they are that the optimum blowing rate for all values of P/D is about 0.5. As P/D is decreased the effectiveness values at all Z/P tend to a narrow band for all M at locations $10 \leqslant X/D \leqslant 30$ and for $P/D = 2.67$ the average values of η in this narrow band is a maximum, about 0.2, for $M \simeq 0.5$. Close to the holes, that is, $X/D < 10$, the centre line effectiveness is greatest for $M < 1.0$ for a row of holes with $P/D = 5.33$ but the average of effectiveness for $M < 1.0$ over a pitch is greatest for a

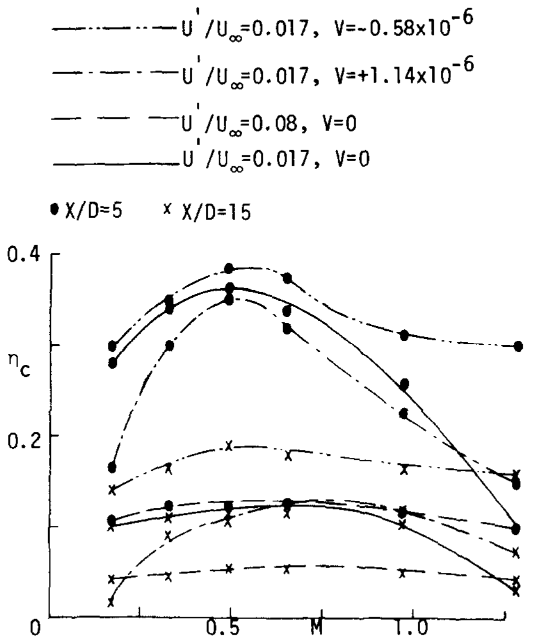

Fig. 5. The influence of free stream turbulence intensity and velocity gradient on film cooling.

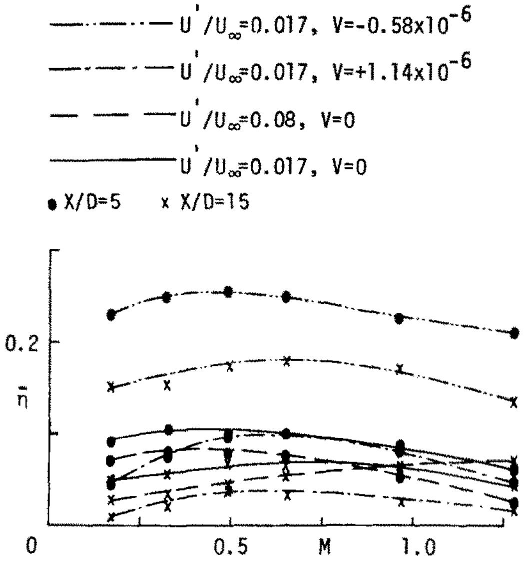

FIG. 6. The influence of free stream turbulence intensity and velocity gradient on film cooling.

row of holes with $P/D = 2.67$. For $M > 1$ the centre line effectiveness close to the holes is about equal for $P/D = 8.0$ and 2.67 and superior to that for $P/D = 5.33$. The average effectiveness for the $P/D = 2.67$ row of holes can be compared at two locations, that is, $X/D = 5$ and $X/D = 15$, with the measurements of Goldstein *et al.* [2], Liess and Carnel [30] and Kruse [5] in Fig. 4. The authors measurements compare favourably with those of other workers.

The above measurements were for free stream turbulence intensity $U'/U_\infty = 0.017$ and zero velocity gradient. It is known that free stream turbulence intensity and velocity gradient influence film cooling performance and in view of this some measurements have been made for $U'/U_\infty = 0.08$ with zero velocity gradient and for velocity gradient factors $V[=(\nu/U_\infty^2)(\mathrm{d}U_\infty/\mathrm{d}x)] = +1.14\times10^{-6}$ and -0.58×10^{-6} with a turbulence intensity of 0.017 with a row of holes of $P/D = 5.33$, see Figs. 5 and 6. It can be seen that higher free stream turbulence intensity lowers the centre line effectiveness for locations $X/D = 5$ and 15 for $M \lesssim 1.25$ whereas the averaged effectiveness is lowered for all M at $X/D = 5$ but it is only lowered for $M \lesssim 0.7$ at $X/D = 15$ and then increased for $M \gtrsim 0.7$. In general the findings on the effect of free stream turbulence intensity agree with those of Brown and Minty [11] for injection through a single square section hole and with Launder and York [10], though their investigation was for favourable free stream velocity gradients. The published work on the effect of free stream velocity gradient on film cooling effectiveness indicates some disagreement on the subject. Liess [14] measured the effect of favourable velocity gradients on film cooling effectiveness downstream from a row of holes and found that the effectiveness was reduced for low values of the blowing parameter, but his work was concerned only with averaged effectiveness. Launder and York [10] found that steep streamwise accelerations substantially improve the effectiveness for discrete hole cooling systems, which was confirmed by the findings of Brown and Minty [11]. The measurements illustrated in Figs. 5 and 6 for injection through a row of holes with $P/D = 5.33$ show that the averaged effectiveness is greatest for adverse velocity gradients and least for favourable velocity gradients for all M and for $X/D = 5$ and 15. For the centre line effectiveness the same is true for $M \lesssim 1.2$ and $X/D = 5$, it is not so for $X/D = 15$ in which case the centre line effectiveness in the

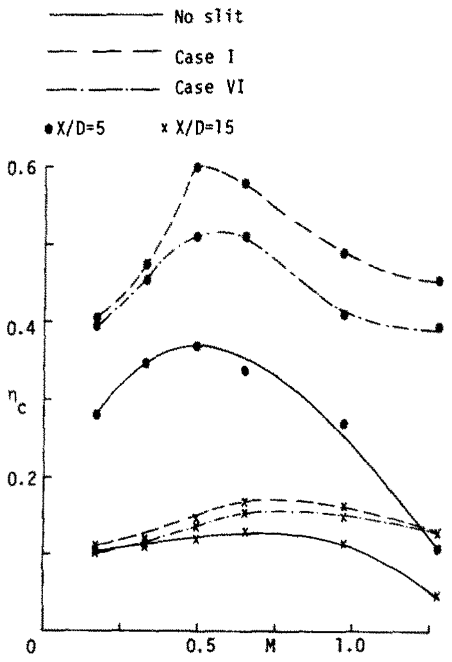

FIG. 7. η_c vs M for fixed values of X/D for a row of holes with $P/D = 5.33$ and slits $3D$ downstream.

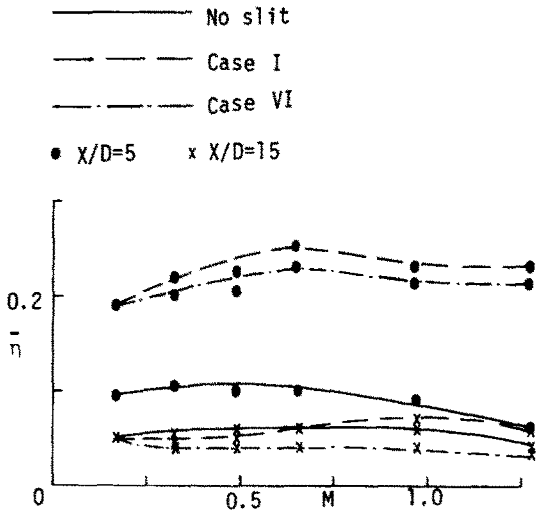

FIG. 8. $\bar{\eta}$ vs M for fixed values of X/D for a row of holes with $P/D = 5.33$ and slits $3D$ downstream.

presence of a favourable velocity gradient although less than that for a zero gradient for $M \lesssim 0.7$ is superior for $M \gtrsim 0.7$. The effect of free stream velocity gradient on film cooling effectiveness would appear to differ from two to three dimensional injection situations.

Interest in the effect of injecting a small amount of coolant through a plate onto its surface a small distance downstream from a row of injection holes was stimulated by effectiveness measurements of Kruse [5]. The coolant arrangement of a row of holes with $P/D = 5.33$ was modified by locating a continuous normal slit for injecting small quantities of coolant onto the surface of the rig base 3D downstream from the trailing edges of the holes, see Fig. 1. The continuous slit could be progressively plugged to create a row of slits of different lengths and pitches, in all six cases were considered going from a continuous slit to a row of slits 2D long, see Fig. 1. For a continuous slit 5% of coolant was passed through the slit and 95% through the row of holes and as the slit was plugged, the percentage of coolant passing through the slits was progressively reduced. Figures 7 and 8 show some typical measurements for this combination cooling arrangement from which it can be seen that close to the holes the centre line and averaged effectiveness are substantially increased with injection of a small quantity of coolant $3D$ downstream from the row of holes for all values of M. At $X/D = 15$ for all injection arrangements the centre line effectiveness is increased for all values of M but there is no significant change in the averaged effectiveness over the situation for injection through a row of holes only.

Three dimensional film cooling from a row of holes is influenced by the fluid mechanic processes involved, Bergeles *et al.* [18, 19]. Some flow measurement and visualisation techniques were used by Saluja [23] to aid in the understanding of the film cooling measurements. It was found that for blowing rates with $M > 0.5$ a separation bubble exists downstream of the injection holes. As the blowing rate increases, the length of the separation bubble increases such that at $M = 0.62$ the bubble extends over a distance of about $3D$. At the blowing rates $M = 0.92$ and 1.24 the jets separate from the surface at the point of injection and do not reattach though the degree of penetration of the jet into the free stream is greater for $M = 1.24$. Turbulence intensity measurements in particular indicate that at $M = 1.24$ the jet has merged with the free stream by the location $X/D = 4$. The lateral spreading of the jets is more apparent for large values of M at locations close to the holes but even for $M \leqslant 0.3$ the velocity and turbulence profiles are similar for all Z/P at $X/D \geqslant 6$. As the free stream turbulence intensity is increased the jets lift off the surface more and spread in the lateral direction earlier. The spreading of the jets is the only desirable aspect of high free stream turbulence intensity and its effect is apparent in the effectiveness results presented above.

DISCUSSION AND CONCLUSIONS ON HEAT SINK MODELS AND CORRELATIONS

In order to use heat sink models it is necessary to evaluate Y_0 and ε. In the present work values of Y_0 were obtained from flow visualisation and hot wire anemometer measurements and the variation of Y_0 with M is given by

$$Y_0/D = 0.46M - 0.13.$$

Though this method gives only approximate values for Y_0 it was acceptable as the maximum value of Y_0/D is about 0.5 and, therefore, neglecting it in the heat sink model equations introduces an error in η of less than 2%. Values for the effective turbulent diffusivity ε were determined from the experimental effectiveness measurements and the variation of Y_0/D with M. The resulting values of ε were found to be functions of the momentum flux ratio I, and location X/D, Z/D, for both the varying strength line sink models and the line sink model of uniform strength covering the complete pitch of holes. This is contrary to the assumption of constant ε in deriving expressions for effectiveness η from heat sink models. From Saluja [23] the parabolically varying strength model was no better than the linearly varying strength model for which

$$\begin{aligned}
\eta(X,Z) = {} & \frac{MU_\infty D}{4\varepsilon(X/D)}\exp\left[-\frac{U_\infty D}{4\varepsilon(X/D)}\left(\frac{Y_0}{D}\right)^2\right]\Bigg[\tfrac{1}{2}(\pi)^{1/2}\left(\frac{2Z}{D}+1\right)\left[\frac{4\varepsilon(X/D)}{U_\infty D}\right]^{1/2}\Bigg(\mathrm{erf}\Bigg\{\tfrac{1}{2}\left[\frac{U_\infty D}{4\varepsilon(X/D)}\right]^{1/2}\left(\frac{2Z}{D}+1\right)\Bigg\} \\
& -\mathrm{erf}\Bigg\{\left[\frac{U_\infty D}{4\varepsilon(X/D)}\right]^{1/2}\left(\frac{Z}{D}\right)\Bigg\}\Bigg)+\frac{4\varepsilon(X/D)}{U_\infty D}\Bigg\{\exp\left[-\frac{U_\infty D}{16\varepsilon(X/D)}\left(\frac{2Z}{D}+1\right)^2\right] \\
& -\exp\left[-\frac{U_\infty D}{4\varepsilon(X/D)}\left(\frac{Z}{D}\right)^2\right]\Bigg\}-\tfrac{1}{2}(\pi)^{1/2}\left(\frac{2Z}{D}-1\right)\left[\frac{4\varepsilon(X/D)}{U_\infty D}\right]^{1/2} \\
& \times\Bigg(\mathrm{erf}\Bigg\{\left[\frac{U_\infty D}{4\varepsilon(X/D)}\right]^{1/2}\left(\frac{Z}{D}\right)\Bigg\}-\mathrm{erf}\Bigg\{\tfrac{1}{2}\left[\frac{U_\infty D}{4\varepsilon(X/D)}\right]^{1/2}\left(\frac{2Z}{D}-1\right)\Bigg\}\Bigg)+\frac{4\varepsilon(X/D)}{U_\infty D} \\
& \times\Bigg\{\exp\left[-\frac{U_\infty D}{16\varepsilon(X/D)}\left(\frac{2Z}{D}-1\right)^2\right]-\exp\left[-\frac{U_\infty D}{4\varepsilon(X/D)}\left(\frac{Z}{D}\right)^2\right]\Bigg\}\Bigg] \qquad (1)
\end{aligned}$$

and

$$\varepsilon(m^2/s) = 0.13\left(\frac{X}{D}\right)^{-0.75} I^{0.56}\exp\left[0.79\left(\frac{Z}{D}\right)\right], \tag{2}$$

where $0.28 \leqslant M \leqslant 1.35$ and $0 \leqslant Z/P \leqslant 1$. The line sink model covering the complete pitch of holes gave

$$\eta(X,Z) = \frac{M(\pi)^{1/2}}{4(P/D)}\left[\frac{U_\infty D}{4\varepsilon(X/D)}\right]^{1/2}\exp\left[-\frac{U_\infty D}{4\varepsilon(X/D)}\left(\frac{Y_0}{D}\right)^2\right] \times\left(\operatorname{erf}\left\{\frac{P}{2D}\left[\frac{U_\infty D}{4\varepsilon(X/D)}\right]^{1/2}\left(\frac{2Z}{D}+1\right)\right\}-\operatorname{erf}\left\{\frac{P}{2D}\left[\frac{U_\infty D}{4\varepsilon(X/D)}\right]^{1/2}\left(\frac{2Z}{D}-1\right)\right\}\right) \tag{3}$$

and

$$\varepsilon(m^2/s) = 0.014\exp[2.53I^{0.5}-0.5(X/D)] \tag{4}$$

where $0.17 \leqslant M \leqslant 0.48$ and $0 \leqslant Z/P \leqslant 0.3$, and

$$\varepsilon(m^2/s) = 0.044\exp[1.52I^{0.5}-0.08(X/D)] \tag{5}$$

where $0.64 \leqslant M \leqslant 1.27$ and $0 \leqslant Z/P \leqslant 0.3$. For lateral locations where $Z/P > 0.3$ it was more difficult to obtain expressions for ε and equations (4) and (5) were less accurate. Comparisons between line sink models and experiment are made in Fig. 9 from which it can be seen that a linearly varying strength line sink model is a good predictor of film cooling effectiveness for a single hole except on the centre line, however, the uniform strength line sink model, at best, has very limited use. It should be noted that equations (2), (4) and (5) are really only applicable to the experimental measurements described in this article, though the trends indicated by these expressions for ε should apply in general.

Correlations for film cooling have been developed by a number of workers based on an energy balance between a two dimensional jet and the free stream and their correlations have been successful for predicting film cooling effectiveness from a continuous slit and a discrete hole. Figure 10 illustrates the usefulness of correlations for predicting the centre line effectiveness for injection through a discrete hole, and superimposed on these figures are the measurements of Goldstein *et al.* [1] and Brown and Minty [11]. The correlations are

$$\eta_c = 0.37[(1-0.65I^{0.5})A_h]^{0.84}, \tag{6}$$

for $0.28 \leqslant M \leqslant 1.35$, and

$$\eta_c = 0.33[(1-0.75I^{0.5})A_h]^{0.71}, \tag{7}$$

for $0.28 \leqslant M \leqslant 1.0$. Also in Fig. 10 the correlation for injection through a single hole given in equation (7) is used to examine the measured values of centre line effectiveness for rows of holes with $P/D = 8.0$, 5.33 and 2.67. It is apparent that equation (7) becomes less useful for correlating measured values of centre line effectiveness for injection through a

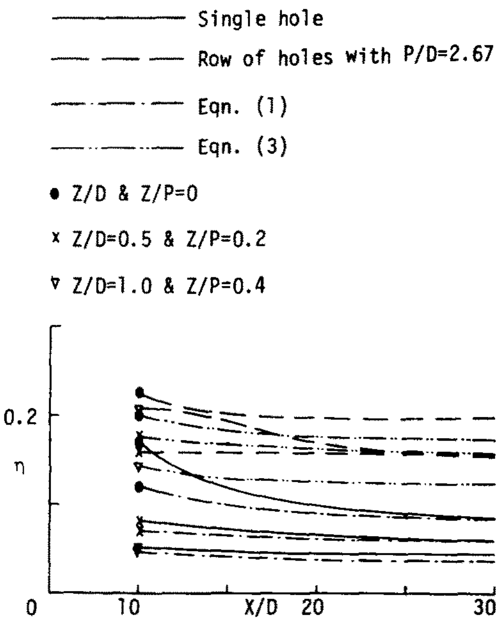

FIG. 9. η vs X/D for fixed values of Z/D for a single hole and Z/P for a row of holes with $P/D = 2.67$ and $M = 0.47$, and compared with equations (1) and (3) respectively.

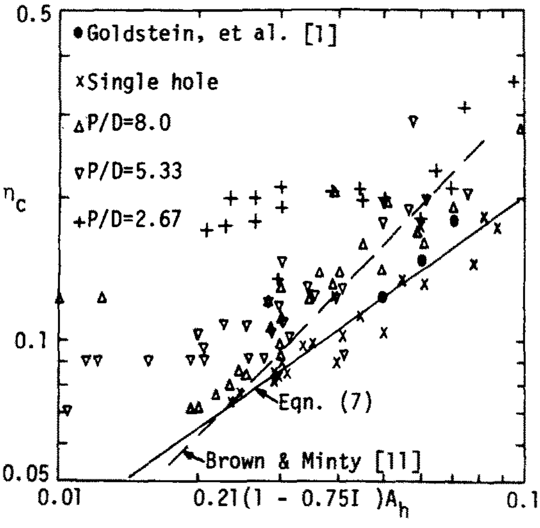

FIG. 10. η_c vs $0.21\ (1-0.75I)A_h$ for a single hole with $0.28 < M < 1.0$, and rows of holes with $P/D = 8.0$, 5.33 and 2.67 and with $0.16 < M < 1.0$.

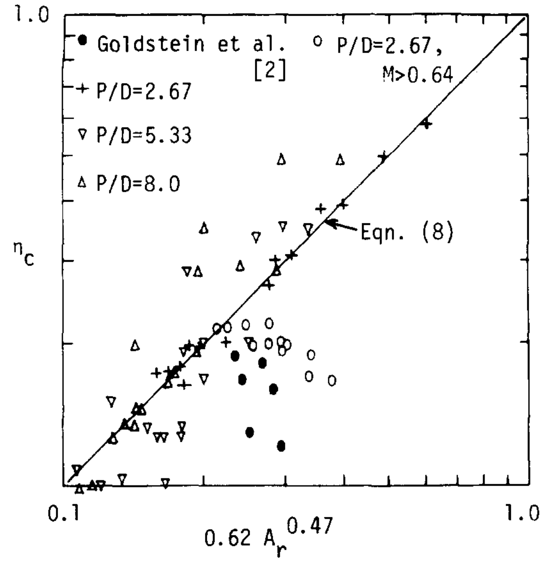

FIG. 11. η_c vs $0.62A_r^{0.47}$ for rows of holes with $P/D = 2.67, 5.33$ and 8.0 and for $0 < M < 0.64$.

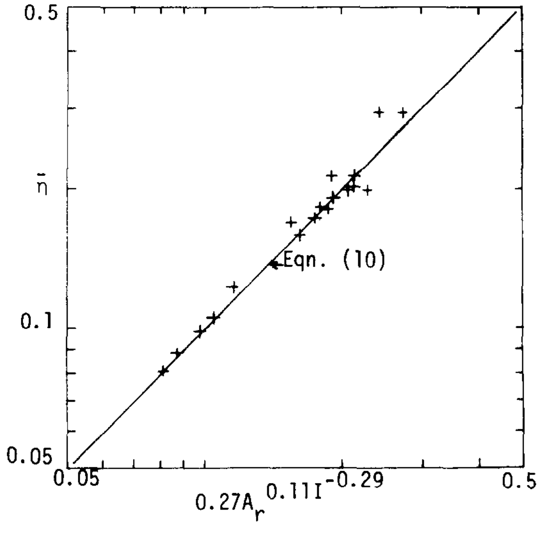

FIG. 12. $\bar{\eta}$ vs $0.27A_r^{0.11I^{-0.29}}$ for a row of holes with $P/D = 2.67$ and for $0 < M < 0.64$.

row of holes as the pitch to diameter ratio decreases and neighbouring jets have progressively more influence on each other.

In many practical situations film cooling with injection through a row of holes is more appropriate than that for a single hole. Correlations for centre line effectiveness and effectiveness averaged over a pitch have been determined for injection through a row of holes with $P/D = 2.67$ and are as follows

$$\eta_c = 0.62A_r^{0.47} \tag{8}$$

for $M \leqslant 0.64$ and

$$\eta_c = 0.23I^{-0.58}A_r^{0.19I^{-0.81}} \tag{9}$$

for $M \geqslant 0.64$, and

$$\bar{\eta} = 0.27A_r^{0.11I^{-0.29}} \tag{10}$$

for $M \leqslant 0.64$ and

$$\bar{\eta} = 0.13A_r^{-0.23} \tag{11}$$

for $M \geqslant 0.64$. The centre line effectiveness measurements for injection through rows of holes with $P/D = 8.0$, 5.33 and 2.67 are plotted against equation (8) in Fig. 11 and the averaged effectiveness measurements for the row of holes with $P/D = 2.67$ are plotted against equation (10) in Fig. 12. Superimposed on Fig. 11 are the measurements of Goldstein *et al.* [2]. From Fig. 11 it can be seen that equation (8) adequately correlates centre line effectiveness for injection through a row of holes with $P/D = 2.67$ and becomes less satisfactory as the pitch to diameter ratio increases. Also, the deviation of the measured results from equation (8) for $M > 0.64$ and the measurements of Goldstein *et al.* [2] for most of which $M > 0.64$ is apparent from Fig. 11. The averaged effectiveness measurements for injection through a row of holes with $P/D = 2.67$ only are plotted in Fig. 12 and these measurements are adequately correlated by equation (10). The fact that a change of correlation occurs at $M \simeq 0.64$ is not surprising in view of the development of separated flow below the jets for $M \gtrsim 0.64$, mentioned earlier.

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REFROIDISSEMENT PAR FILM A PARTIR D'UNE OUVERTURE ET D'UNE LIGNE DE TROUS AVEC DIFFERENTS RAPPORTS PAS SUR DIAMETRE

Résumé—On présente des mesures d'efficacité de refroidissement par injection à travers une ouverture unique et une ligne de trous avec des rapports pas sur diamètre égaux à 8,0, 5,33 et 2,67, avec un angle d'injection de 30° par rapport à la direction de l'écoulement libre. Dans le cas d'une ligne de trous avec un rapport pas sur diamètre de 5,33, on étudie l'influence sur le refroidissement par film de l'injection d'une petite fraction de réfrigérant à travers des fentes étroites situées à trois diamètres en aval de la ligne de trous et on la trouve importante. On mesure aussi les effets de la turbulence de l'écoulement libre et du gradient de vitesse; en général l'accroissement de la turbulence réduit l'efficacité du refroidissement par film, tandis que l'influence du gradient de vitesse est différente pour plusieurs situations d'écoulement. Les mesures sont analysées à partir de modèles de puits linéaires et de formules basées sur le bilan d'énergie; ces dernières décrivent mieux les mesures d'efficacité du refroidissement par film.

FILMKÜHLUNG MITTELS EINER EINZELBOHRUNG UND MIT EINER BOHRUNGSREIHE BEI UNTERSCHIEDLICHEM VERHÄLTNIS VON TEILUNG ZU DURCHMESSER

Zusammenfassung—Es wird das Ergebnis von Messungen zur Wirksamkeit von Filmkühlung bei Kühlmitteleinspritzung durch Einzelbohrungen und durch Bohrungsreihen mit einem Verhältnis von Teilung zu Durchmesser von 8,0; 5,33 und 2,67 mitgeteilt. Der Einspritzwinkel beträgt 30° gegenüber der Richtung der freien Strömung. In Fall der Bohrungsreihe mit einem Verhältnis von Teilung zu Durchmesser von 5,33 wird der Einfluß auf die Filmkühlung untersucht, wenn ein geringer Teil des Kühlmittels durch enge Spalte eingespritzt wird, die sich drei Durchmesser stromabwärts von der Bohrungsreihe befinden. Es wird ein ausgeprägter Einfluß dieser Anordnung festgestellt. Die Auswirkungen der Turbulenz und des Geschwindigkeitsgradienten der freien Strömung werden ebenfalls gemessen. Im allgemeinen vermindert eine erhöhte Turbulenz der freien Strömung die Wirksamkeit der Filmkühlung, wogegen sich der Einfluß des Geschwindigkeitsgradienten der freien Strömung bei zwei- und dreidimensionalen Strömungszuständen unterscheidet. Die Messungen werden über ein Liniensenkenmodell sowie mit Korrelationen, die sich auf eine Energiebilanz stützen, ausgewertet. Die letzteren sind für Darstellung der Meßergebnisse weit besser geeignet.

ПЛЁНОЧНОЕ ОХЛАЖДЕНИЕ ПРИ ПОДАЧЕ ЖИДКОСТИ ИЗ ЕДИНИЧНОГО ОТВЕРСТИЯ И РЯДА ОТВЕРСТИЙ С РАЗЛИЧНЫМ ОТНОШЕНИЕМ ШАГА К ДИАМЕТРУ

Аннотация — Представлены результаты измерения эффективности плёночного охлаждения при вдуве охладителя через единичное отверстие и ряд отверстий с отношениями шага к диаметру, равными 8,0; 5,33 и 2,6 при подаче жидкости под углом 30° к основному потоку. Для ряда отверстий с отношением шага к диаметру, равным 5,33, исследовано влияние на эффективность плёночного охлаждения вдува малой доли охладителя через узкие щели, расположенные на расстоянии трёх диаметров вниз по потоку от местоположения отверстий, и найдено, что это влияние является существенным. Измерено также воздействие турбулентности основного потока и градиента скорости и найдено, что вообще увеличивающаяся турбулентность снижает эффективность охлаждения, в то время как градиент скорости основного потока оказывает различное влияние в зависимости от того, является ли поток двух- или трёхмерным. Проведен анализ результатов измерений с помощью линейных моделей стока и зависимостей, основанных на балансе энергий. Последние наиболее пригодны при обработке результатов измерений эффективности плёночного охлаждения.